Semi-empirical and Analytical Model Development to Predict the Temperature Distribution within the Substrate during the Cold Spray Deposition Process

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Introduction

Speed of the Gas Jet and the Particle in Cold Spraying

- Plastic deformation of the particles
- Final quality of the coating \((Papyrin \, 2006)\)
- Increasing the gas pressure might be difficult or expensive

Pre-heating the Propellant Gas Jet

- Increases the velocity of the particles \((Irissou, \, et\, al. \, 2008 \, & \, Schmidt, \, et\, al. \, 2009)\)
- Increases the in-flight particles’ temperature \((Fukumoto, \, et\, al. \, 2007)\)
- Increases the substrate temperature \((Legoux, \, et\, al. \, 2007 \, & \, Watanabe, \, et\, al. \, 2015)\)

But

- Cause detrimental thermal stress in the coating
- Damage temperature sensitive materials
Introduction

Motivation

- Study the gas jet/substrate heat exchange
- Determine the substrate/coating temperature distribution

- Notable heat transfer rate as a result of pre-heating the gas
- Gas/substrate heat exchange needs further analytical study
- Forced convection is dominant
- Heat transfer coefficient is the focal point

(Modified figure from Donaldson & Snedeker, 1971)
Objectives

- Developing a semi-empirical analytical model to:
  - Determine the heat transfer coefficient of a stationary impinging jet
  - Estimate the transient temperature distribution within the substrate

- Studying the effect of the:
  - Nozzle stand-off distance
  - Time that the nozzle remains stationary

- Comparing the analytical and experimental results
Experimental Methods
# Experimental Method

## Cold spray unit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>660 KPa</td>
</tr>
<tr>
<td>Gas Temperature</td>
<td>300°C</td>
</tr>
<tr>
<td>Stand-off distance</td>
<td>15 - 100 mm</td>
</tr>
</tbody>
</table>

## Substrate information

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Al 6061-T6</td>
</tr>
<tr>
<td>Thickness</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Diameter</td>
<td>20 cm</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>167 W/m-K</td>
</tr>
</tbody>
</table>

## Insulation information

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Vitreous aluminosilicate</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>0.2 W/m-K</td>
</tr>
</tbody>
</table>

(Modified figure from McDonald et al. 2012)
Experimental Method

Measurements

• Measurements by Infrared camera (FLIR A65)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>IR resolution</td>
<td>640 × 512 pixels</td>
</tr>
<tr>
<td>Image frequency</td>
<td>9 Hz</td>
</tr>
<tr>
<td>Object temperature range</td>
<td>-40°C to 160°C</td>
</tr>
<tr>
<td>Accuracy</td>
<td>±5°C or ±5% of reading</td>
</tr>
</tbody>
</table>

• Surface of the substrate painted in black
Mathematical Model
Mathematical Model

- Transient 2D Heat Conduction Equation

\[ \frac{1}{r} \left( r \frac{\partial}{\partial r} \left( r \frac{\partial T(r,z,t)}{\partial r} \right) \right) + \frac{\partial^2 T(r,z,t)}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T(r,z,t)}{\partial t} \]  \hspace{1cm} (1)

- Initial and boundary conditions

\[ T(r = 0, z, t) = \text{Finite} \]  \hspace{1cm} (2)

\[ \frac{\partial T(r = b, z, t)}{\partial r} = 0 \]  \hspace{1cm} (3)

\[ -k \frac{\partial T(r, z = 0, t)}{\partial z} = h(r)\left[T_{aw}(r) - T(r, z = 0, t)\right] \]  \hspace{1cm} (4)

\[ -k \frac{\partial T(r, z = \delta, t)}{\partial z} = 0 \]  \hspace{1cm} (5)

\[ T(r, z, t = 0) = T_0 \]  \hspace{1cm} (6)
Mathematical Model

- Green’s function technique used to solve the transient heat conduction problem

- Simplified Green’s function solution for boundary condition of third kind:

\[
T(r, z, t) = \int_{r'=0}^{b} \int_{z'=0}^{\delta} G(r, z, t | r', z', \tau) \cdot F(r', z') r'dz'dr' \cdot \alpha_s \int_{\tau=0}^{t} d\tau \int_{r'=0}^{b} \int_{z'=0}^{\delta} G(r, z, t | r', z', \tau) \cdot \frac{1}{k_s} f(r', z', \tau) r'dr' \quad (7)
\]

- Using Separation of Variables method for the problem with homogeneous boundary condition to find the Green’s function:

\[
G(r, z, t | r', z', \tau) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{\exp[-\alpha_s (\lambda_i^2 + \beta_j^2)(t-\tau)]}{N(\lambda_i)N(\beta_j)} J_0(\lambda_i r) J_0(\lambda_i r') \\
\times [\tan(\beta_j \delta) \sin(\beta_j z) + \cos(\beta_j z)] [\tan(\beta_j \delta) \sin(\beta_j z') + \cos(\beta_j z')]
\]
Mathematical Model

Final equation to determine the temperature distribution

\[
T(r,z,t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \left[ T_0 \frac{\exp[-\alpha_s (\lambda_i^2 + \beta_j^2) t]}{N(\lambda_i)N(\beta_j)} J_0(\lambda_i r)[\tan(\beta_j \delta) \sin(\beta_j z) + \cos(\beta_j z)] \right]
\times \int_{r'=0}^{b} \int_{z'=0}^{\delta} J_0(\lambda_i r')[\tan(\beta_j \delta) \sin(\beta_j z') + \cos(\beta_j z')] r' dz' dr'
\]

\[+ \left( \frac{\alpha_s}{k_s} \right) \left[ \frac{h(r)}{N(\lambda_i)N(\beta_j)} \right] J_0(\lambda_i r)[\tan(\beta_j \delta) \sin(\beta_j z) + \cos(\beta_j z)]
\times \frac{1}{\alpha_s (\lambda_i^2 + \beta_j^2)} \left[ 1 - \exp(-\alpha_s (\lambda_i^2 + \beta_j^2) t) \right] \int_{r'=0}^{b} J_0(\lambda_i r') T_{aw}(r') r' dr' \].

\[
N(\lambda_i)N(\beta_j) = \int_{r=0}^{b} \int_{z=0}^{\delta}[\tan(\beta_j \delta) \sin(\beta_j z) + \cos(\beta_j z)]^2 [J_0(\lambda_i r)]^2 r dr dz.
\]

\[
\begin{cases}
\lambda_i J_1(\lambda_i b) = 0 & \text{Eigenvalues in radial direction} \\
\beta_j \tan(\beta_j \delta) - h(r)/k = 0 & \text{Eigenvalues in axial direction}
\end{cases}
\]
Results and Discussion
Results and Discussion

Adiabatic Wall Temperature

- To consider the effects of the dissipative heat release in the air film

\[ \psi = \frac{\text{SOD}}{D_n} \]

(Modified figure from Donaldson & Snedeker, 1971)
Results and Discussion

Nusselt Number

- Heat Transfer Coefficient and Heat Flux (different times)
Nusselt Number

- Comparison at different stand-off distances (Fo=27)

(Modified figure from Zuckerman & Lior, 2006)
Validation of the Semi-empirical Analytical Model

- Non-dimensional surface temperature at different stand-off distances

![Graphs showing non-dimensional surface temperature at different stand-off distances for SOD = 15 mm and SOD = 25 mm.](image-url)
Validation of the Semi-empirical Analytical Model

- Non-dimensional surface temperature at different stand-off distances

\[ \theta_s = \frac{(T_s - T_\infty)}{(T_g - T_\infty)} \]

\[ \eta = \frac{r}{D_n} \]

SOD = 50 mm

SOD = 100 mm
Conclusions

- Analytical method to determine heat transfer coefficient for impinging air jet during a cold-gas dynamic spraying was studied.

- Heat transfer coefficient was found to be time-independent.

- It was further found that the maximum values for the non-dimensional adiabatic wall temperature and the Nusselt number were present in the vicinity of the stagnation point of the air jet, which agreed with observations of other studies.

- Non-dimensional surface temperature at different stand-off distances and different time steps were in good agreements with the experimental results.
Thank You

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