CHAPTER 6

Total Strain-Based Strain-Range Partitioning—Isothermal and Thermomechanical Fatigue

THE STRAIN-RANGE PARTITIONING (SRP) method deals primarily with how creep and plastic inelastic strains are reversed in tension and compression during a strain cycle. In the problems of the preceding chapters (Chapters 3 to 5), the inelastic strains in each half of a cycle were generally large and could be deduced either by direct measurement or computation. However, when these strains are quite small, their indirect determination may be preferable. This is certainly the case for thermal-induced strains. Here, it may be possible to determine the total imposed strains involved because they are directly related to constraint of easily perceived thermal expansions. However, the mechanical strain components are usually much smaller and are mainly elastic. Hence, separating the inelastic from the elastic strains becomes much more difficult, although various procedures are available.

This chapter describes procedures to separate inelastic from the elastic strains. Two basic approaches are discussed. The first method directly calculates the inelastic strain. The other method reformulates the SRP method into a total strain-range approach in which the elastic and inelastic strain ranges are combined to form a total strain range. This method is somewhat analogous to the Universal Slopes Equation approach presented in Chapter 3, “Fatigue Life Relations,” of the companion volume (Ref 6.1). The life equations are formulated to deal with the total strain range rather than with its individual elastic and inelastic components.

Direct Determination of Inelastic Strain-Range Components

Although the SRP method is detailed in preceding chapters of this book, some basic discussions are repeated here to demonstrate the unique considerations required for application to problems involving small strains. The first approach, described subsequently, is taken directly from the early work of the authors (Ref 6.2) in 1977. The second approach referred to as the total strain version of SRP, is be described in a later section.

Basic Data Required. In order to treat creep-fatigue problems SRP, it is desirable to know a number of properties associated with the material and with the particular problem being treated. Usually, the information will be readily available through basic material characterization, but even if not known accurately, it may be possible to estimate the required quantities with reasonable accuracy. In the discussion to follow, we shall assume that the required information is available for large as well as small strains. If small-strain information is difficult to obtain directly, it will be assumed that the determination is made by simple extrapolation from high strain data. We shall illustrate the procedure in connection with problems of moderately high-strain range because these are the only ones for which good experimental data are available. Then, we shall show results of calculations involving small strains, although there are, of course, no
extensive data available to check the validity of these calculations. Figure 6.1 shows the ideal type of database desired for life analysis by this method. The basic inelastic strain range versus cyclic life relationships for \( \Delta \varepsilon_{\text{pp}}, \Delta \varepsilon_{\text{CC}}, \Delta \varepsilon_{\text{CP}}, \) and \( \Delta \varepsilon_{\text{pc}} \)-type strain ranges are shown in Fig. 6.1(a). They are shown as mildly temperature-dependent functions; although at least two materials (316 stainless steel and 214Cr-1Mo steel) studied (Ref 6.3) were found to be essentially independent of temperature. This figure also illustrates an elastic component \( (\Delta \varepsilon_{\text{el}, \text{pp}}) \) obtained from rapid cycling in association with \( \Delta \varepsilon_{\text{pp}} \) tests. These lines are temperature-dependent, reflecting the flow strength and elastic modulus dependency of the material with temperature.

Figure 6.1(b) shows the cyclic stress-strain curve \( OA \) for rapid cycling of the material. There is, of course, interdependence between \( OA \) and the inelastic (PP) and elastic (\( \Delta \varepsilon_{\text{cp}, \text{pp}} \)) lifelines of Fig. 6.1(a). For any selected life value, the elastic line can be used to determine the stress range, and the elastic and inelastic lines together can be used to determine the total strain range. Thus, in principle, the curve \( OA \) can be constructed from knowledge of the life relationships. In this approach, the implication is that plastic strain is always present, even at very low stresses that appear to lie on the linear (elastic) portion of the curve, but the deviation from linearity is very small. It is advantageous to regard a PP strain to be present at all stress ranges because it enables the determinations of plasticity strains, even though small, when treating low total strains.

Also shown in Fig. 6.1(b) is the rapid-cycling hysteresis loop \( ABCDA \) for one strain range. Presuming that there are no phase transformations due to temperature, this loop can be constructed from the shape of the cyclic stress-strain curve through application of the well-known double-amplitude construction principle (discussed in Chapter 2, “Stress and Strain Cycling,” of the companion volume, Ref 6.1). That is, \( CDA \) can be constructed from knowledge of \( OA \) by choosing \( C \) as the origin and doubling all stress and strain values along \( OA \). Similarly, \( ABC \) is symmetrical to \( CDA \). Although only one cyclic stress-strain curve and hysteresis loop is shown in Fig. 6.1(b), numerous hysteresis loops could be drawn, one loop at each strain range. All of these curves and loops are sensitive to temperature, being a reflection of the rheological dependence on temperature. Once the life relationships for the inelastic and elastic strain ranges (for PP cycling) in Fig. 6.1(a) are known for a selected temperature, any required hysteresis loop in Fig. 6.1(b) can be constructed.

The third desirable ingredient in the SRP formulation is shown in Fig. 6.1(c). It is the relationship between stress and secondary (steady-state) creep rate from a cyclic creep test. Alternatively, it is the stress versus steady-state creep rate relationship obtained during cyclic tests involving stopping at specific stress levels and observing the creep rate once the hysteresis loop had been cyclically stabilized (see the section “Experimental Partitioning of Creep and Plastic Strains” in Chapter 5 of this book). It is commonly recognized that the Bailey-Norton power law equation represents the relationship between secondary creep rate \( \varepsilon_{\text{c}} \), and stress, \( \sigma \). Thus, Fig. 6.1(c) shows linear plots of stress and creep rates on log-log coordinates. These curves can be expected to be strongly dependent on temperature. For illustrative purposes, the lines for different temperatures are shown parallel, but other relationships are possible.

The final ingredient of the life analysis is shown in Fig. 6.1(d). It is a stabilized hysteresis loop for a typical duty cycle under analysis. Although an experimentally determined hysteresis loop is highly desirable, it is not necessary. In its absence, it can be approximated from other specified variables (for example, linear stress ramping, as later discussed, or other pattern of stress or strain variation). However, it should be emphasized that hysteresis loops stabilize rapidly, and it is usually necessary to traverse only a small fraction of life expectancy in order to obtain this valuable adjunct to the analysis.

**Outline of Procedure.** To illustrate the procedure, we shall analyze a test of the alloy 316 stainless steel reported by Conway et al. (Ref 6.4) involving strain-hold to long-times (1.0 h/ cycle). The strain range for these tests was large (2%), but it will be seen that exactly the same procedure as discussed for this high-strain problem can be used to analyze low-strain problems. In fact, results will be shown for such calculations after the method is described.

The ingredients analogous to Fig. 6.1 applicable to this problem are shown in Fig. 6.2. Since the test was conducted at a constant temperature of 650 °C (1200 °F), only this temperature is reflected in Fig. 6.2(b) to (d). Figure 6.2(a) shows the strain pattern imposed at the test condition, and Fig. 6.2(e) is the stress-strain response, showing the stress relaxation that was measured.
during the strain-hold period. The time markings on Fig. 6.2(e) correspond to those in Fig. 6.2(f).

The analysis is shown in Fig. 6.3. First, the secondary creep strain is calculated in each half of the cycle. Since the compressive half of the cycle involves only rapid loading, the creep is negligible; only the tensile half involves creep. Step (a) shows the creep rates and the integrated area (representing the total creep strain) as 0.000975 for the 60 min tensile hold period. In step (b), it is also shown that the plastic strain range is 0.0160, as deduced from the stress range (elastic strain range) and the life relationships of Fig. 6.2(a). The total inelastic strain range is 0.0167, as deduced from the width of the hysteresis loop, from which the transient creep strain range is determined by subtracting the plastic strain range. The calculated strain ranges differ but slightly from the measured values reported by Conway et al. (Ref 6.4).

It then becomes possible to calculate the strain-range components as discussed in Chapter 1, “Creep Under Monotonic and Cyclic Loading,” in this book. The “creep” in each half-cycle consists of the secondary creep plus 10% of the transient creep, if identified. Since, in this case, the transient creep is known, the total tensile creep, step (e), is 0.001045. Now, since the “creep” in the compressive half of the cycle is zero, there can be no reversed creep;

![Figure 6.1](image-url)

**Fig. 6.1** Input information for treating creep fatigue by strain-range partitioning. (a) Partitioned strain-range life relationships. (b) Cyclic stress-strain curve and hysteresis loop for rapid cycling obtained by principle of double-amplitude construction. (c) Relationship between steady-state creep rate and stress. (d) Hysteresis loop for a cycle of interest. Source: Ref 6.2
Fig. 6.2 Input information for analysis of hold-time test. (a) Strain-time history. (b) Strain-range life curves. (c) Cyclic stress-strain curve. (d) Relationship between steady-state creep rate and stress. (e) Hysteresis loop with various tensile hold times. (f) Stress relaxation curve during hold time. Source: Ref 6.2
thus, $\Delta \varepsilon_{CC} = 0$. All the tensile “creep” is reversed by plasticity; thus, $\Delta \varepsilon_{CP} = 0.001045$. The remainder of the inelastic strain range is converted to reversed plasticity, $\Delta \varepsilon_{PP}$, and is $0.015655$, according to step (i). Thus, this problem involves $\Delta \varepsilon_{CP}$ and $\Delta \varepsilon_{PP}$, and, as shown in steps (j) and (k), results in a computed life of 155 cycles. This compares to a measured life of 103 cycles, which is a reasonably close correlation. Similar calculations were made for the other two 2% strain-range hold-time tests reported by Conway in Ref 6.4. In one, the hold-time was 60 min and the other 30 min. Predicted lives were 156 and 149 cycles, compared to the experimental lives of 117 and 76, respectively.

**Alternate Procedure.** Before presenting the calculations for small strains, it is appropriate to describe an alternate procedure for handling the stress relaxation problem just discussed. In the previous discussion, it was assumed that the stress pattern during the relaxation is known from experimental observation. Suppose, however, that experimental determination is inconvenient; can we still handle the problem? Initially, let us assume that the maximum stress is known from the strain range and cyclic stress-strain curve. That is, in Fig. 6.4 we assume that $RP$ follows the cyclic stress-strain curve (by the double-amplitude rule of stress and strain, Fig. 6.1(b). Recognizing the large strain range of 2% involved in this problem, we reasonably assume that the stresses at $R$ and $P$ will be approximately equal in magnitude; thus, there is no ambiguity as to the coordinates of point $R$ in the initial construction of the hysteresis loop. The stress $\sigma_p$ at point $P$ will be known in magnitude. The stress pattern $PQ'Q$ can then be determined from a single creep relaxation analysis as follows.

Letting $\varepsilon_c$ be the creep strain at any time $t$ after the initiation of hold, and also letting $\varepsilon_{el}$ be the relaxed elastic strain at this time and $\sigma$ be the relaxed value of stress at this time, then, neglecting primary (transient) creep, and considering only the secondary creep rate, from Eq 6.1:

$$\dot{\varepsilon}_c = \frac{d\varepsilon_c}{dt} = A\sigma^n$$

This is the Bailey-Norton power-law relationship shown earlier. The material constants $A$ and $n$ are temperature-dependent. By Hooke’s law:

$$\frac{d\sigma}{E} = \frac{d\varepsilon_e}{dt} = \frac{1}{N_f} \frac{d\varepsilon}{dt}$$

However since strain is held constant:

$$\frac{de}{dt} = 0, \quad \frac{d\sigma}{dt} = -\frac{d\varepsilon}{dt}$$

Combining Eq 6.1 to 6.3 results in:

$$\frac{d\sigma}{\sigma^n} = -AE\delta t$$

Integrating Eq 6.4 from $t = 0$ to $\delta$:

$$\left(\frac{\sigma_{\pi}}{\sigma^{n+1}}\right)^{-n+1} \left(\frac{\sigma_{\pi}}{\sigma^{n+1}}\right)^{-n+1} = -AE[t - 0] = -AEt$$

or:

$$\sigma_{\pi} = \left[\sigma_{\pi}^{-n+1} + (n-1)AEt\right]^{-\frac{1}{n+1}}$$

(Eq. 6.6)
After the entire hold-period \( \delta_t \), the relaxed stress becomes:

\[
\sigma_\varphi^{\prime} = \left[ \sigma_P^{-n+1} + (n-1)AE\delta t \right]^{-\frac{1}{n+1}} \quad \text{(Eq 6.7)}
\]

For the problem illustrated in Fig. 6.2, the application of the double-amplitude cyclic stress-strain relationship results in a stress \( \sigma_P \) of 41.87 ksi, and since the constants in the creep (Eq 6.1) are known from National Aeronautics and Space Administration (NASA) data (Ref 6.5) and unpublished data obtained during preparation of Ref 6.5:

\[ n = 7.14 \]
\[ A = 2.55 \times 10^{-17} \] (for \( \sigma \) in ksi, \( \delta t \) in seconds)
\[ E = 22 \times 10^6 \text{ ksi} \]
Thus:

\[
\sigma_t = \left[ \sigma_p^{0.14} + 3.44 \times 10^{-12} \Delta \right]^{1/61}
\]

(Eq 6.8)

A plot of stress relaxation according to Eq 6.8 is shown in Fig. 6.5(a); the agreement is remarkably good considering the basic approximations involved and the fact that the creep-rate determinations were made by different investigators and on different lots of material from those involved in the relaxation tests, and that transient creep was omitted.

To check the validity of this approach to other tests, additional calculations were made for the two other relaxation tests reported in Ref 6.4. The temperature and strain range were the same as the aforementioned, but in one case, the hold time was 30 min and in the other, only 1 min. The results are shown in Fig 6.5(b) and (c). For the 30 min hold-time test, the agreement is still very good. For the 1 min hold-time, the effect of the transient creep is apparent in the early seconds, but after approximately 30 sec, the agreement again becomes excellent.

The next step is to determine how much of each type of strain-range component develops during the cycle, as described subsequently. Figure 6.6 shows a summary of calculations made by the following procedure for three additional tests taken from Ref 6.4. The predictions agree well with the experiments. We also note that the degree of agreement between prediction and experiment does not decrease as the total test time increases.

The total amount of plastic flow during the tensile half is known from the cyclic stress-strain curve, or as expressed by the linear life relationships, shown in Fig. 6.4(c). Knowing the stress range \( RP \), the elastic strain range establishes the point \( M \) on the elastic lifeline, from which the point \( N \) on the plastic lifeline vertically above \( M \) establishes a plastic strain range of 0.0160.

The tensile creep is equal to the elastic strain from \( P \) to \( Q \) (or, alternatively, the integrated creep during \( PQ \) according to Eq 6.1, which yields exactly the same result). Thus, in this case, the tensile creep strain is 0.000975.

The determination of the compressive plasticity may involve a small amount of ambiguity if compressive plasticity is determined from consideration of the shape of the curve \( QR \). We cannot construct this reverse piece of the hysteresis loop from the double-amplitude cyclic stress-strain curve alone, starting with \( Q \) as an

![Fig. 6.5](http://www.asminternational.org) Comparison of experimental and calculated stress relaxation using power-law equation between stress and secondary creep rate (a) 60 min. (b) 30 min. (c) 1 min. Source: Ref 6.2
The plastic flow from Q to R, based only on the stress range involved, would be expected to be too small to balance both the tensile plastic and tensile creep flow.

Actually, a more appropriate way to construct the compressive half of the loop is to add the imaginary segment PTQ in Fig. 6.4(a) so that TQR, in conjunction with the cyclic stress-strain curve, yields a compressive plastic flow that is equal to the tensile plastic flow plus the relaxation creep flow. This adds the complication that if \( \sigma_T = \sigma_r \), then \( \sigma_p \) is no longer equal to, \( \sigma_s \), which was our original premise.

Therefore, the problem becomes one of trial and error to determine the appropriate location of the hysteresis loop to establish consistency with the rheological behavior. In general, the rheological behavior, as affected by the cycle itself, influences the individual creep and elasticity components. This is evident in Fig. 6.5, wherein it is noted that the maximum tensile stress reached depends on the hold-time. The longer the hold-time, the lower the stress. To obtain exact behavior, the constitutive equations must be better established than they are now. Nevertheless, the approximate answer can readily be determined in this case by stating that the behavior of QR in the vicinity of R is such that the plastic flow developed on compression is equal to the tensile plastic flow plus the tensile creep flow during PQ.

Thus, from the earlier considerations, the \( \Delta \varepsilon_{PP} \) deformation is equal to the tensile creep deformation 0.000975, and the remainder of the inelastic strain is \( \Delta \varepsilon_{PP} \), so that \( \Delta \varepsilon_{PP} = 0.0160 \). The life is then calculated by the Interaction Damage Rule:

\[
F_{PP} = \frac{0.0160}{0.016975} = 0.943 \quad F_{CP} = \frac{0.000975}{0.016975} = 0.057
\]

From Fig 6.1(a):

\( N_{PP} = 233 \quad N_{CP} = 26 \)

And from the Interaction Damage Rule:

\[
\frac{1}{N_f} = \frac{F_{PP}}{N_{PP}} + \frac{F_{CP}}{N_{CP}} = \frac{0.943}{233} + \frac{0.057}{26} = 0.00624
\]

\( N_f = 160 \)

This calculated life compares with the measured value of 103 cycles, which is well within the commonly accepted factor of 2. Note that this calculation made no use of experimentally determined hysteresis loops or stress relaxation patterns. The calculations are still satisfactory, despite the neglect of transient creep, although the inclusion of transient creep would improve the predictions because the value of \( F_{CP} \) would be greater.

**Extension to Treatment of Very Low Inelastic Strains**

We now extend the same concepts already described in connection with the treatment of large strains to the study of low inelastic strains. The elements of the procedure are:

- The determination of the plastic strain range from knowledge of the stress range and the life relationships of elastic and plastic strain ranges
- The determination of secondary creep strains by integrating the equations relating creep rate to a power-law of stress
- Determining transient (or primary) creep strains from actual observations of total creep during any interval, and subtracting the secondary creep strains. This step is optional and is omitted if cyclic creep experiments

![Fig. 6.6 Correlation of observed and predicted cyclic lives for tensile strain hold-time tests. (a) Correlation with respect to cyclic lives. (b) Correlation of cyclic lives with respect to hold-time. Source Ref 6.2](image-url)
fatigue and durability of metals at high temperatures

are unavailable or if a semiexperimental phase is inconvenient.

- Constructing the strain-range components—\( \Delta e_{PP} \), \( \Delta e_{PC} \), \( \Delta e_{CC} \) from the determined creep and plasticity components in the tensile and compressive halves of the cycle
- Applying the Interaction Damage Rule to determine life

Of special importance is the determination of the stress values that develop. To this extent, the stresses will be known accurately either if directly measured by experimental observation of the hysteresis loop or if accurate constitutive equations are available to track stress and strain during the cycle. However, in some cases, neither approach will be practical; then, the calculations will involve engineering approximations. We shall illustrate a case in which such approximations are required, their choice being made to introduce some conservatism in the resulting life estimates.

**Combined Tensile and Compressive Hold Periods.** We first treat the case in which both tension and compression hold periods are introduced in problems involving low strain range. It will be seen that this case lends itself more readily to the estimation of the hysteresis loop developed because of the symmetry of the cycle. Figure 6.7 illustrates the procedure. We start with the recognition that because of the symmetry of the loading cycle, the hysteresis loop will be symmetrical in the tensile and compressive halves. Then, the hysteresis loop will be ABCD for the strain range \( \Delta e \). Thus, there is only one unknown quantity in this analysis, for example, \( \sigma_B \). Once we know \( \sigma_B \) and the hold-time, we can determine \( \sigma_C \) from the relaxation Eq 6.7, and, of course, \( \sigma_D \) and \( \sigma_A \) follow from considerations of symmetry. Because of the nonlinearity of the problem, however, it is convenient to start with the assumption of known stresses and to determine the combinations of strain ranges and hold-times that will generate these stresses. For example, in Fig. 6.7, suppose we were concerned with the solution for a total strain range \( \Delta e_T = 0.5\% \) and various hold-times. A convenient approach is first to construct a complete hysteresis loop \( MABNCMDM \) for an arbitrarily selected strain range, say 1% under rapid cycling. This can readily be done from the cyclic stress-strain curve \( \times 2 \) (or the log-log linear life relationships).

Vertical lines \( AD \) and \( BC \) can then be constructed at equal distances from the vertical axis and at a strain range of 0.5%, to determine the specific stress values \( \sigma_A \), \( \sigma_B \), \( \sigma_C \), and \( \sigma_D \). It can then be immediately determined what hold time \( \delta_t \) is required to relax \( \sigma_B \) to \( \sigma_A \) (or \( \sigma_D \) to \( \sigma_B \)). Since the partitioning of the hysteresis loop \( ABCD \) into creep and plasticity components can readily be accomplished (even when curvature is present along \( AB \) and \( CD \)), the strain-range components are easily established. Of course, because of symmetry, only \( \Delta e_{PP} \) and \( \Delta e_{CC} \) develop, and, in fact, for small \( \Delta e \), the inelastic strain range developed is almost entirely \( \Delta e_{CC} \). In either case, the life can readily be calculated from the Interaction Damage Rule. Thus, the calculation provides one point relating the life to \( \Delta e_T \) and \( \delta t \). Additional points can be obtained from the same loop \( MN \) by selecting a new value of \( \Delta e_T \) and repeating the procedure to determine a new hold-time and life. In a similar manner, by choosing a new value of \( \Delta e_T \) (say 0.5%), and proceeding with a spectrum of choices of \( \Delta e \), a new series of corresponding values of hold-time and life values can be computed.

The calculations can then be depicted in their entirety, as shown in Fig. 6.8 and Fig. 6.9. These figures are discussed later, after presenting results for calculations involving hold-times only in tension or only in compression.

**Tensile Hold Periods.** Treatment of only tensile hold problems is not as straightforward as symmetrical tensile and compressive holds, because of ambiguities that develop in the rheological behavior at low stress ranges. Consider, for example, the two extremes of behavior possible when a specimen is cycled at a small strain

![Hysteresis loop construction to simplify analyses of symmetrical hold-time tests. Source: Ref 6.2](image-url)
range and tensile hold periods are introduced. In Fig. 6.10(a), the behavior is depicted as involving little or no plasticity during the reversal because of the small strain range involved. In the first straining, the stress-strain path is along OA.

Although, according to the procedure adopted herein, all stress applications imply plastic strain (e.g., see Fig. 6.4c), the actual amount of plasticity is negligibly small if the total strain range is small enough; therefore, we show line OA as a straight line. If the hold period at maximum strain is long enough, the stress can relax to point B. Thus, upon reverse straining, the path is BC that we again assume to be at a strain range low enough to preclude significant plastic defor-

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**Fig. 6.8** Creep-fatigue lives predicted for cyclic total strain ranges from 2% to 0.05% for symmetric hold-times from 0.1 to 1000. Strain range is the parameter associated with each solid line. Dashed line represents a total time to failure of 30 years. Source: Ref 6.2

**Fig. 6.9** Results of predictions from Fig 6.8 shown with hold-time per cycle as the parameter. Source: Ref 6.2

**Fig. 6.10** Two possible extremes of behavior in strain cycling at low strain range with tensile strain hold-times. (a) Ratcheting resulting in eventual shakedown, wherein no cyclic inelastic strain develops. (b) Eventual development of closed hysteresis loop with cyclic inelastic strains. Source: Ref 6.2